Χρήσιμες (και μη) Σχέσεις από «Ταλαντώσεις και Κύματα

$\Psi(x,t) = A\sin 2\pi [\frac{x}{\lambda} - \frac{t}{T}]$, $\upsilon_\sigma = -\frac{2\pi A}{T}\cos 2\pi [\frac{x}{\lambda} - \frac{t}{T}]$, $\frac{\partial \Psi(x,t)}{\partial t} = \mp\upsilon\frac{\partial \Psi(x,t)}{\partial x}$,

$\frac{\partial^2 \Psi(x,t)}{\partial t^2} = \upsilon^2 \frac{\partial^2 \Psi(x,t)}{\partial x^2}$, $\Psi(x,t) = B\sin kx \cos\omega t$, $\Psi(x,t) = A\sin(kx - \omega t)$,

$y(t) = A\sin(\omega t + \varphi)$, $y(t) = A\sin(2\pi\nu t + \varphi)$, $m\frac{d^2 y}{dt^2} = -D\,y$, $\omega = \sqrt{\frac{D}{m}}$,

$y(t) = C_1\sin(\omega t) + C_2\cos(\omega t)$, $W = \int_{x_1}^{x_2} -Dx dx = \frac{1}{2}Dx_1^2 - \frac{1}{2}Dx_2^2$,

$E = \frac{1}{2}Dx^2 + \frac{1}{2}m\upsilon^2$, $E = \frac{1}{2}DA^2$, $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, $Q = \frac{\omega_0}{\gamma}$

Λύση Χαρακτηριστικής Εξίσωσης $m\rho^2 + b\rho + D = 0$	**Γενική Λύση Διαφορικής Εξίσωσης** $m\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + D\,x(t) = 0$
1η περίπτωση : $\Delta \equiv b^2 - 4mD > 0$ 2 πραγματικές ρίζες ρ_1 , ρ_2	$x(t) = C_1 e^{\rho_1 t} + C_2 e^{\rho_2 t}$ **Φθίνουσα Κίνηση**
2η περίπτωση : $\Delta \equiv b^2 - 4mD < 0$ 2 μιγαδικές ρίζες $\rho_r \pm i\rho_i$, $\rho_r = -b/2m$	$x(t) = e^{\rho_r t}[C_1\cos(\rho_i t) + C_2\sin(\rho_i t)]$ **Ταλαντωτική Κίνηση με Απόσβεση ή «Φθίνουσα Ταλάντωση»**
3η περίπτωση: $\Delta \equiv b^2 - 4mD = 0$ 1 διπλή πραγματική ρίζα $\rho = -b/2m$	$x(t) = C_1 t e^{\rho t} + C_2 e^{\rho t}$ **Φθίνουσα Κίνηση. Κρίσιμη Απόσβεση**

Σύγκριση Μηχανικών και Ηλεκτρικών Ταλαντωτών

$q \Leftrightarrow x$, $V(x,t) \Leftrightarrow F(x,t)$, $I = \frac{dq}{dt} \Leftrightarrow \upsilon = \frac{dx}{dt}$, $R \Leftrightarrow \frac{F}{\upsilon}$, $\hat{Z}_\mu \Leftrightarrow \hat{F}/\upsilon$,

$$\hat{Z}_\mu = b + i\left(\omega m - \frac{D}{\omega}\right) = b + iR_{mD} \Leftrightarrow \hat{Z}_{\eta\lambda} = R + i\left(\omega L - \frac{1}{\omega C}\right) = R + iR_{LC} ,$$

$$Z_\mu e^{i\varphi} = Z_\mu(\cos\varphi + i\sin\varphi) \qquad Z_{\eta\lambda}e^{i\varphi} = Z_{\eta\lambda}(\cos\varphi + i\sin\varphi)$$

$$Q = \frac{m}{b}\sqrt{\frac{D}{m}} = \frac{1}{b}\sqrt{mD} \Leftrightarrow Q = \frac{L}{R}\sqrt{\frac{1}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$, $x(t) = \frac{F_0\cos(\omega t - \delta)}{m[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$,

$\delta = \arctan\frac{\gamma\omega}{\omega_0^2 - \omega^2}$, $A(\omega) = \frac{F_0/m}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$, $\omega_{\max} = \sqrt{\omega_0^2 - \frac{\gamma^2}{2}}$,

$$\frac{d^2x_1}{dt^2}+\omega_0{}^2x_1=-\frac{D}{m}(x_1-x_2)\,,\ \frac{d^2x_2}{dt^2}+\omega_0{}^2x_2=-\frac{D}{m}(x_2-x_1)\,,\ X_+=x_1+x_2$$

$$X_-=x_1-x_2\ ,\ \frac{d^2X_+}{dt^2}+\omega_0{}^2X_+=0\ ,\ \frac{d^2X_-}{dt^2}+(\omega_0{}^2+2\frac{D}{m})X_-=0\,,\ (\omega_{0,+})^2=\omega_0{}^2\,,$$

$$(\omega_{0,-})^2=\omega_0{}^2+2\frac{D}{m}\ ,\ x_1(t)=\frac{1}{2}[A_+\cos(\omega_{0,+}t+\delta_+)+A_-\cos(\omega_{0,-}t+\delta_-)]\,,$$

$$x_2(t)=\frac{1}{2}[A_+\cos(\omega_{0,+}t+\delta_+)-A_-\cos(\omega_{0,-}t+\delta_-)]\,,\ (\hat{D}-\varepsilon\hat{I})\vec{A}=0\ ,\ \left|\hat{D}-\varepsilon\hat{I}\right|=0\,,$$

$$F(x,t)=Ce^{i\lambda x}e^{-\kappa x}e^{-i\omega t}\ ,\ F(x,t)=(A\sin\lambda x+B\cos\lambda x)e^{-\kappa x}\cos\omega t\ ,$$

$$\Psi(\vec{r},t)=Ae^{i(\vec{k}\vec{r}-\omega t)}=Ae^{i(k_xx+k_yy+k_zz-\omega t)}\,,\ \Psi(r,t)=\frac{Ae^{i(kr-\omega t)}}{r}\,,n^2(\omega)=1+\frac{Ne^2}{\varepsilon_0m}\cdot\frac{1}{(\omega_0{}^2-\omega^2)}$$

$$\begin{matrix}\vec{E}\Rightarrow\vec{E}_d\\ \varepsilon_0\Rightarrow\varepsilon\end{matrix}\ ,\ -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}\ ,\ \begin{matrix}y_1(x,t)=A_1\sin(kx-\omega t+\varphi_1)\\ y_2(x,t)=A_2\sin(kx-\omega t+\varphi_2)\end{matrix}\ ,$$

$$\begin{matrix}y_1(x,t)=A_1\sin(kx+\omega t+\varphi_1)\\ y_2(x,t)=A_2\sin(kx+\omega t+\varphi_2)\end{matrix}\,,\Psi(x,t)=y_1(x,t)+y_2(x,t)\,,\Psi(x,t)=A\sin[\omega t+\delta(x,\varphi)]\,,$$

$$A^2=A_1{}^2+A_2{}^2+2A_1A_2\cos(\delta_2-\delta_1)\,,\tan\delta=\frac{A_1\sin\delta_1+A_2\sin\delta_2}{A_1\cos\delta_1+A_2\cos\delta_2}\,,$$

$$I=I_1+I_2+2\sqrt{I_1I_2}\cos(\delta_2-\delta_1)\,,\qquad y_0(x,t)\equiv2A\cos\left(\frac{\Delta kx}{2}-\frac{\Delta\omega t}{2}\right)\,,$$

$\Delta x=0,\pm\lambda,\pm2\lambda,\cdots$ ή $\Delta x=\pm n\lambda\,,n=0,1,2,3,$ ***<u>Συνθήκη ενισχυτικής συμβολής.</u>***

$\Delta x=\pm(2n+1)\frac{\lambda}{2}\,,n=0,1,2,3,$ ***<u>Συνθήκη καταστροφικής συμβολής.</u>***

ή $d\sin\varphi=\pm n\lambda\ ,\ n=0,1,2,3,\cdots$ ***<u>Συνθήκη πρωταρχικών μεγίστων ,</u>***

$d\sin\varphi=\pm\frac{m\lambda}{N}\,,m=1,2,\cdots,N-1$ ***<u>Συνθήκη ελαχίστων .</u>***

$d\sin\varphi=\pm\frac{(m+\frac{1}{2})\lambda}{N}\ ,m=1,2,\cdots,N-2$ ***<u>Συνθήκη δευτερευόντων μεγίστων.</u>***

$$\begin{matrix}(\Delta x)(\Delta k)\geq1\\ (\Delta t)(\Delta\omega)\geq1\end{matrix}\ \text{ή}\ \begin{matrix}(\Delta x)(\Delta k)\geq2\pi\\ (\Delta t)(\Delta\omega)\geq2\pi\end{matrix}$$ ***<u>Σχέσεις Αβεβαιότητος .</u>***

$$Z=\rho\upsilon\,,Z_{HM}\sim\frac{1}{n}\,,R=\frac{B_1}{A_1}=\frac{Z_1-Z_2}{Z_1+Z_2}\,,T=\frac{A_2}{A_1}=\frac{2Z_1}{Z_1+Z_2}\,,R_{HM_{12}}=\frac{n_1-n_2}{n_1+n_2}\,,Z_2=\sqrt{Z_1Z_3}$$

$$A_ie^{i[k_il_{xi}x-\omega t]}+A_re^{i[k_r(l_{xr}x+l_{yr}y)-\omega t]}=A_te^{i[k_t(l_{xt}x+l_{yt}y)-\omega t]}\,,\ I(\theta)=I_0\frac{\sin^2\gamma}{\gamma^2}\cdot\frac{\sin^2(N\beta)}{\sin^2(\beta)}\ ,$$

$$L=\frac{\lambda_2}{4}=\frac{\lambda}{4n_2}\ ,\theta_i=\theta_r\,,\ \frac{\sin\theta_1}{\sin\theta_2}=\frac{\lambda_1}{\lambda_2}=\frac{n_2}{n_1}\ ,\ I=I_0\frac{\sin^2\gamma}{\gamma^2}\ ,\ \sum_{i=1}^{N}n_is_i\Rightarrow\int_{\Sigma}^{\Sigma'}n(s)ds\,.$$

$$\boxed{\sin\theta=m\frac{\lambda}{\alpha}\quad(m=\pm1,\pm2,\pm3,...)}$$