2) Ηλεκτρόνιο ακίνητο μέσα σε $\vec{B}(t) = B_0 \sin(\omega t)\hat{z}$

Για $t=0$ $S_n \psi_{(t=0)} = \frac{\hbar}{2}\psi_{(t=0)}$

$$S_n = \vec{S}\cdot\hat{n} = \frac{1}{\sqrt{2}}S_x + \frac{1}{\sqrt{2}}S_y = \frac{\hbar}{2\sqrt{2}}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}$$

$\hat{n} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\psi_{(t=0)} = \begin{pmatrix} a \\ b \end{pmatrix}$ άρα $\frac{\hbar}{2\sqrt{2}}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow$

$$\Rightarrow \frac{1-i}{\sqrt{2}} b = a \Rightarrow e^{-i\frac{\pi}{4}} b = a$$

$$\frac{1+i}{2} a = b \Rightarrow e^{i\frac{\pi}{4}} a = b \qquad |a|^2 + |b|^2 = 1$$

$$\psi(t=0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\frac{\pi}{4}} \end{pmatrix}$$

$$H_B \psi_{(t)} = i\hbar \frac{\partial \psi}{\partial t} \Rightarrow (-)\left(-\frac{ge}{2m_e}\right) B_0 \sin\omega t \, S_z \psi = -i\hbar\frac{\partial\psi}{\partial t}, \; \psi = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$H_B = -\vec{\mu}_{spin}\vec{B} = (-)\left(\frac{-ge}{2m_e}\right)\vec{S}\cdot\vec{B}$$

$$\frac{geB_0\hbar}{4m_e}\sin\omega t\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} a \\ b \end{pmatrix} = i\hbar\begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix}, \quad \omega_0 = \frac{geB_0}{4m_e}$$

$$\omega_0 \hbar \sin\omega t \, a = i\hbar\frac{da}{dt} \Rightarrow -i\omega_0\int_0^t \sin(\omega t)\,dt = \int_{a_0}^{a}\frac{da}{a} \Rightarrow$$

$$-\omega_0\hbar\sin\omega t \, b = i\hbar\cdot\frac{db}{dt} \qquad \Rightarrow -i\frac{\omega_0}{\omega}(1-\cos\omega t) = \ln\left(\frac{a}{a_0}\right)$$

άρα $a = a_0 \cdot e^{i\frac{\omega_0}{\omega}(\cos\omega t - 1)}$ άρα $\psi_{(t)} = \begin{pmatrix} a_0 e^{i(\sim)} \\ b_0 e^{i(\sim)} \end{pmatrix}$

$b = b_0 e^{i\frac{\omega_0}{\omega}(\cos\omega t - 1)}$ Για $t=0$ $a_0 = \frac{1}{\sqrt{2}}$

$$b_0 = \frac{e^{i\pi/4}}{\sqrt{2}}$$

$P_i = |\langle \chi_{(x)} | \Psi_{(t)} \rangle|^2 = \dots$ $\chi_-(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(β) $\chi_+(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

άρα $P_{(z)} = |\langle \chi_+ | \Psi_{(t)} \rangle|^2 = \dots = \frac{1}{2}$

(4) $\Psi(\vec{r}, t=0) = N \underset{n\ell m}{\Psi_{211}} \chi_+$, $\vec{B} = B_0 \hat{z}$, $t > 0$

$H = H_0 + H_B$

(β) $P(+\frac{\hbar}{2}) = 1$ (επειδή η Ψ έχει μόνο χ_+)

$P(-\frac{\hbar}{2}) = 0$

(γ) $\vec{J} = \vec{L} + \vec{S}$

Ξέρουμε ότι $|\ell - \frac{1}{2}| \leq j \leq \ell + \frac{1}{2} \Rightarrow \frac{1}{2} \leq j \leq \frac{3}{2}$ άρα $j = \frac{1}{2}, \frac{3}{2}$

(δ) $\langle \Psi | \Psi \rangle = 1 \Rightarrow N = 1$

$\Psi_{211} = R_{21}^{(r)} Y_{11}(\theta, \varphi)$

(α) $H\Psi = \dots$

$$H = H_0 + H_B = H_0 - \frac{e}{2m_e} \vec{L}\vec{B} - \frac{ge}{2m_e} \vec{S}\vec{B} = H_0 - \frac{eB_0}{2m_e} L_z - \frac{ge}{2m_e} S_z$$

$$H\Psi(\vec{r}, t=0) = (H_0 \Psi_{211})\chi_+ - R_{21}\left(\frac{eB_0}{2m_e} L_z Y_{11}\right)\chi_+ - R_{21} Y_{11}\left(\frac{geB_0}{2m_e} S_z \chi_+\right)$$

$$H \cdot \Psi = \left(E_2^{(0)} - \frac{eB_0}{2m_e}\hbar - \frac{geB_0}{2m_e}\frac{\hbar}{2}\right)\Psi = E\Psi \quad , \quad \Psi_{(t)} = \Psi_{(t=0)} e^{-i\frac{Et}{\hbar}}$$

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Λύσεις Κβάντο 2

1) $F=-kx$, $\psi(x)=Nxe^{-\lambda x^2/2}$

$\quad\; -\frac{dV}{dx} \longrightarrow V=\frac{1}{2}kx^2$ Για να έχει καθορισμένη ενέργεια πρέπει $H\psi=E\psi$

(a) $H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)$, $k=m\omega^2$

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2 \quad \text{αλλά} \quad H\psi=-\frac{\hbar^2}{2m}\psi''+\frac{1}{2}m\omega^2x^2\psi$$

αλλά $\psi'=Ne^{-\lambda x^2/2}-\lambda Nx^2e^{-\lambda x^2/2}$

$\psi''=(-3\lambda+\lambda^2x^2)\psi$

άρα $-\frac{\hbar^2}{2m}(-3\lambda+\lambda^2x^2)\psi+\frac{1}{2}m\omega^2x^2\psi=E\psi$

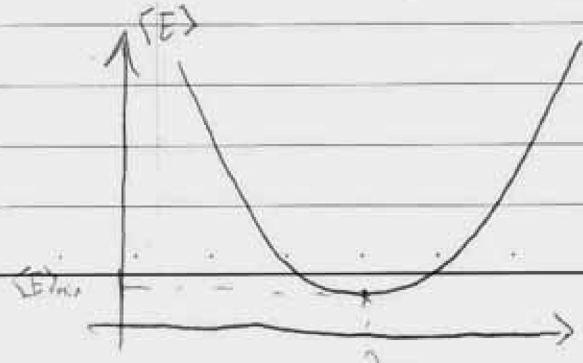
Για να είναι ιδιοσυνάρτηση πρέπει (να ~~...~~ μηδενίζονται οι δυνάμεις του x^2)

(1) $-\frac{\hbar^2}{2m}\lambda^2+\frac{1}{2}m\omega^2=0 \Rightarrow \lambda^2=\frac{m^2\omega^2}{\hbar^2} \Rightarrow \lambda=\frac{m\omega}{\hbar}$

(2) $\frac{3\hbar^2}{2m}\lambda=E \Rightarrow E=\frac{3}{2}\hbar\omega$

(β) $$\langle E\rangle=\int_{-\infty}^{\infty}\psi^*H\psi\,dx=\int\psi^*\left\{-\frac{\hbar^2}{2m}(-3\lambda+\lambda^2x^2)\psi+\frac{1}{2}m\omega^2x^2\psi\right\}dx=$$

$$=\frac{3\lambda\hbar^2}{2m}\underbrace{\int_{-\infty}^{\infty}\psi^*\psi\,dx}_{=1}+\frac{1}{2}\left(m\omega^2-\frac{\lambda^2\hbar^2}{2m}\right)\int\psi^*x^2\psi\,dx=\frac{3\hbar^2\lambda}{4m}+\frac{3m\omega^2}{4\lambda}$$



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