(1)

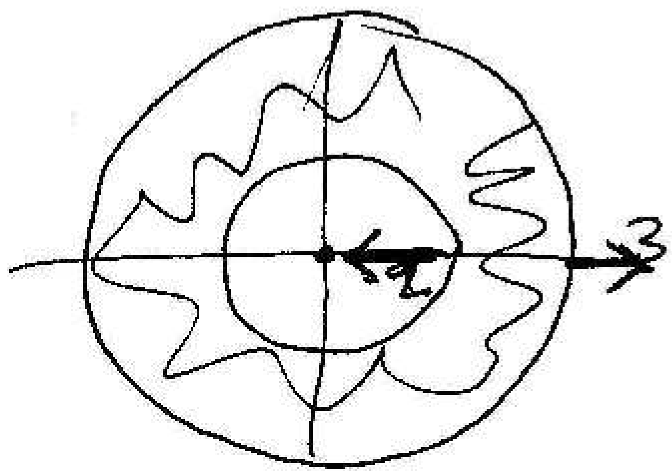
$$\Delta u(\rho,\phi) = 2\rho\cos 2\phi \quad 2<\rho<3 \quad 0\le\phi<2\pi$$

$$\frac{\partial u}{\partial \rho}(2,\phi) = A + \sin\phi \qquad \frac{\partial u}{\partial \rho}(3,\phi) = 0$$

Πότε είναι επιλύσιμο??

$$\int_{\Omega} \Delta u \, d\underline{x} = \int_{\Omega} \nabla\cdot\nabla u \, d\underline{x} = \int_{\partial\Omega} \hat{n}\cdot\nabla u \, ds =$$

$$= \int_{\partial\Omega} \frac{\partial u}{\partial n} dS$$



$$\int_2^3\int_0^{2\pi} \underbrace{\Delta u(\rho,\phi)}_{2\rho\cos 2\phi} \rho \, d\rho\, d\phi = \int_0^{2\pi} 0\cdot 3\, d\phi - \int_0^{2\pi} (A+\sin\phi)\, 2\, d\phi$$

$$\Delta w = w_{\rho\rho} + \frac{1}{\rho} w_\rho + \frac{1}{\rho^2} w_{\phi\phi}$$

$$\Delta u(\rho,\phi) = 2\rho\cos 2\phi$$

$$\left.\frac{\partial u}{\partial \rho}\right|_{\rho=2} = \sin\phi \qquad \left.\frac{\partial u}{\partial \rho}\right|_{\rho=3} = 0$$

$$u = w + v \qquad \Delta w = 0 \qquad \Delta v = 2\rho\cos 2\phi$$

$$v_{\rho\rho} + \frac{1}{\rho} v_\rho + \frac{1}{\rho^2} v_{\phi\phi} = 2\rho\cos 2\phi$$

Αναζητώ λύση της μορφής

$$U = A\rho^3 \cos 2\varphi$$

Έστω $\quad U = \frac{1}{5}\rho^3 \cos 2\varphi$

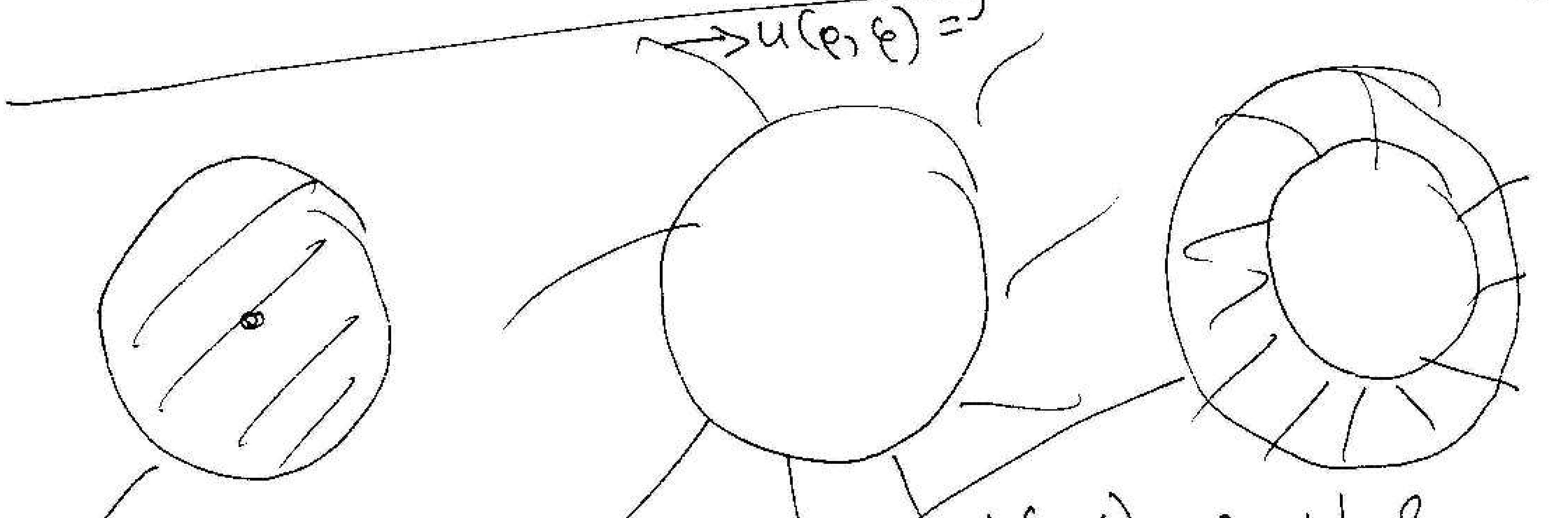
Έχω να λύσω το πρόβλημα

$$\boxed{\Delta w = 0}$$

$$\left.\frac{\partial w}{\partial \rho}\right|_{\rho=2} = \frac{\partial u}{\partial \rho} - \left.\frac{\partial U}{\partial \rho}\right|_{\rho=2} = \sin\varphi - \frac{1}{5}2^3\cos 2\varphi$$

$$\left.\frac{\partial w}{\partial \rho}\right|_{\rho=3} = \frac{\partial u}{\partial \rho} - \left.\frac{\partial U}{\partial \varphi}\right|_{\rho=3} = 0 - \frac{1}{5}3^3\cos 2\varphi$$

$$u(\rho,\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \rho^{-n}(a_n \cos n\varphi + b_n \sin n\varphi)$$



$$u(\rho,\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \rho^{n}(a_n \cos n\varphi + b_n \sin n\varphi)$$

$$u(\rho,\varphi) = a_0 + b_0 \ln\rho + \sum_{n=1}^{\infty}\left[(a_n\rho^n + b_n\rho^{-n})\cos n\varphi + (c_n\rho^n + d_n\rho^{-n})\sin n\varphi\right]$$

$$\Delta u(x_1, x_2) = \underline{\underline{6}} \quad \text{Poisson}$$

$$\Delta u(x_1, x_2, t) = u_{tt} \quad (\text{Κυματική})$$

$$\boxed{u_{x_1x_1} + u_{x_2x_2} = u_{tt}}$$

$$u_{xx} = \frac{1}{a^2} u_t$$