There are three conserved quantities in Mechanics: Linear Momentum, Energy, and Angular Momentum. These are not the only conserved quantities that physicists have discovered but they are the three that Mechanics predicts will be conserved for systems of particles interacting via forces with a simple set of characteristics. I will now go on to describe each quantity in turn and the assumptions that must be made about interaction forces that will lead to conservation.

First and foremost there is **Linear Momentum**. In Newtonian Mechanics linear momentum is a quantity associated to the motion of particles. The formula for it is simple to write down. The linear momentum of a particle is its mass times its velocity. Since velocity is a vector so is linear momentum and it points in the same direction as the velocity of the particle. Linear Momentum was so central to Newton's understanding of mechanics that he called it the "quantity of motion" in a particle. The linear momentum of a system of particles is just the sum of the momenta of all of the particles. Why is linear momentum conserved in mechanics? The answer is Newton's Symmetry principle of interactions, his 3rd Law. This law implies that when any two particles interact that the CHANGES in the linear momentum of the two particles are exactly opposite. This means that the (vector) sum of the two linear momenta cannot be changed by the interaction. The only effect on linear momentum that interactions can have is transfer linear momenta between particles. In fact, the force acting on a particle is <u>exactly</u> the rate at which linear momentum is being transferred by that interaction. [We also have a special name for the total linear momentum change caused by an interaction. It is called the IMPULSE delivered by the force.] This gives us a whole new conceptual way to view the effects of interactions (!) and it doesn't end there. Because the total linear momentum is just the simple vector sum of the linear momenta of all the particles in the system it is possible to play a simple mathematical trick that reveals that the total linear momentum of any system of particles can also be thought of as the total mass of the system times a weighted average of the velocities of all the particles in the system. [The weighting is by the fraction of the total mass that each particle carries.] Following the idea even further one can see that this velocity is actually THE VELOCITY of a particular point associated to every system of particles. This point is called the **Center of Mass** of the system. This point is located at the mass weighted average of all the positions of the particles in the system. One more mathematical step shows that this point moves with exactly the acceleration a particle would have if it had a mass equal to the total mass of the system and was subjected to only the EXTERNAL forces being exerted due to interactions of the system with other particles on the outside. Take a moment to think about this because it is a major piece of intellectual magic. This means that as long as Newton's 3rd Law is true, it follows that every system of particles behaves in an essential way just like a single particle!! This is the reason why it often doesn't matter how we divide a real system into "particles". No matter how you cut it, there is a point attached to the system that responds to external influences like an abstract "particle". This is the most significant source of the flexibility that one has in modelling systems in Newtonian Mechanics. This is not to say that all interesting questions about the behavior of a system of particles can be answered with models that treat the whole system as a single particle. Not by a long shot. But it does say that if a system stays in "one piece" and one is only concerned with the motion of the system on scales comparable to its size and larger, then modelling it as a single particle will always work! I can only assume that Newton was deeply aware of these facts and like any good mathematician made sure that the symmetry of interactions that guarantees these results was incorporated as a fundamental principle of all interactions in his paradigm.

The second conserved quantity is **Energy**. ***It is important to understand that it is possible to build models in mechanics that predict the motion of particles correctly but that do NOT conserve energy.*** This point is sometimes lost on beginning students and at first blush appears to be in conflict with the sweeping statement that energy is a conserved quantity in Mechanics. The precise situation is this: IF all the interactions in a model are moderated by Conservative Forces, i.e. described by force laws with a particular property (explained below) then the total energy of a closed system will not change in time. A particular model that we build may fail to conserve energy basically for two reasons. Sometimes the system we are modelling is NOT closed in the sense that we often build models of a subsystem of a larger system with a variety of internal interactions and also the external influences due to the interaction of this subsystem with its environment. So it is possible that the energy in the subsystem is actually NOT constant because it is exchanging energy with its environment. These models capture the real physical situation and we do NOT expect the energy of such subsystems to be constant because it is simply not the case. A more subtle and interesting problem with energy arises when we model composite objects as a single particle. To understand this one must understand how we count up the total energy in a system.

There are two kinds of energy in Newtonian mechanics: <u>particle energy</u> (more often called KINETIC ENERGY) and <u>interaction energy</u> (more often called POTENTIAL ENERGY). Particle energy is a property of particles and depends only on the mass and speed of each particle. The little formula for this is $1/2\ m\ v^2$; where m is the mass and v is the speed. This formula makes it clear that this kind of energy is associated with the motion of particles where the faster they go the more KINETIC energy they have. So the particles in a system carry a certain amount of energy at a moment depending on their speeds (and masses). Obviously the kinetic energy of particles can be changed by the action of forces. If the aggregate force on a particle speeds it up over some time period then some energy has been delivered to the particle and if the particle is slowed down then some of its kinetic energy has been removed. The amount of energy transferred to or from a particle by a particular interaction is called the WORK done on the particle and the amount of energy depends on the average component of the force along the direction of the particle's velocity. The average in this case is the distance weighted average. Particles must move through space in order for some work to be done. The rate of energy transfer is called the POWER. For the purpose of this discussion it is important to notice that particle energy is always positive or zero, never negative. This implies that when we add up all the particle energies in a system that there can be no cancellations, in contrast with linear momentum. A collection of particles vibrating or with random velocities can have ZERO total linear momentum but will always have a positive kinetic energy count. A sizable amount of energy can be stored in particle energy at a microscopic level and it will never be seen in the gross motion of the body because the total linear momentum is negligibly small. If we model such a system of particles as a single particle then our model will completely ignore this energy content. Energy can hide inside a system of particles in another way as well.

It turns out that while the symmetry of interactions (Newton's 3rd Law) guarantees that any linear momentum depleted from one particle is instantly delivered to the other particle in the interaction pair, this symmetry does <u>NOT</u> guarantee that the same can be said for the particles' energies. [In fact it is IMPOSSIBLE to invent a force law that will only transfer energy AND momentum between otherwise unconstrained particles!] What can happen is that an interaction can conserve all the (particle) linear momentum in a system and still act to take away or deliver energy to the particles; but in doing so the interaction itself MUST, at least temporarily, store some of the energy itself!! This is a profound realization because it implies that the interactions themselves can carry the (conserved, immutable, transcendent, tangible!) physically measurable ENERGY themselves. The total energy obtained by adding up all the particle energies AND interaction energies in a closed system IS conserved. The (symmetric!) interactions with this property are called CONSERVATIVE and are completely described by formulas that give the interaction energy as a function of the <u>relative</u> positions of the particles. When we say that Energy is a conserved quantity in Mechanics we are making a strong assumption: that there is a level of subdivision of systems into particles where all the interactions are accurately modelled with conservative forces. This assumption is often true in the sense that for a wide range of phenomena all the observed changes in the kinetic energies, as well as all the changes in their momenta can be accounted for using conservative forces. When the assumption has proven false at some level of particle identification this has prompted physicists to look for internal structures or undetected interactions that will account for any apparent spontaneous generation or absorption of energy. At the level of classical mechanics all we can say is that IF all the controlling interactions are conservative then the combined Kinetic and "Potential" energy of a closed system is conserved. Famous examples of conservative forces are (Newtonian) Gravitational and Electrostatic Interactions. These two interactions alone can be used to model a wide range of fundamental behavior. The search for force laws that describe conservative forces is aided by the existence of a simple mathematical test, a computation, that allows one to check to see if a proposed force formula describes a conservative interaction.

The last of the three conserved quantities in Mechanics is **Angular Momentum**. Like linear momentum, Angular Momentum is a vector quantity associated strictly to particles in Mechanics. Like energy, it is only strictly conserved when the interactions have a particular property in addition to symmetry. The property of interactions that will guarantee that Angular Momentum is conserved for a closed system is that the interaction forces be directed along the line joining the positions of the particles. Such forces are called Central Forces. Angular Momentum is a measure of the circulation of mass about a point in mechanics. If the point is the center of mass of the system then the associated angular momentum is called the intrinsic angular momentum or the SPIN of the system. The angular momentum of a system of particles about an arbitrary fixed point can always be decomposed into the (vector) sum of two pieces: the SPIN and the ORBITAL angular momentum. The orbital angular momentum is the just the angular momentum one would calculate by assuming that the whole system is a single particle located at the center of mass of the system. The names SPIN and ORBITAL are historical and come from the analysis of angular momentum of the planets in motion about the sun. Each planet carries orbital angular momentum about the sun and spin angular momentum from its rotation about its axis. The relatively rapid rotational motion one sees in the heavens, orbital motion of planets around the sun, stars around the center of galaxies, and the spinning of stars, planets, and moons on their axes, can be understood as manifestations of the concentration of angular momentum in relatively compact objects due to their collapse from extended objects under the influences of the (central!) force of gravity. As in the case of the other two conserved quantities, we have a way of calculating the rate that an interaction transfers this conserved quantity to or from a particle and a special name for it. The rate that a force changes the angular momentum of a particle is called the TORQUE. [Angular Momentum is often given short shrift in introductory course on Mechanics because it is the least intuitive and most mathematically complicated to describe of the three conserved mechanical quantities. I will try to avoid doing this in our course.]

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