EXAMPLE 4.14

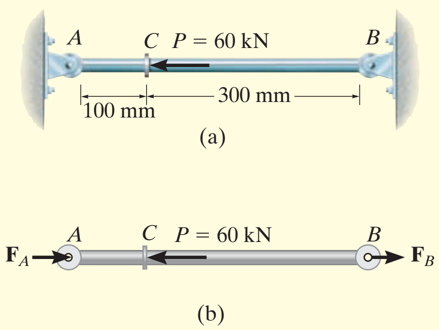


Fig. 4–30

The rod shown in Fig. 4–30*a* has a radius of 5 mm and is made of an elastic perfectly plastic material for which $\sigma_Y = 420$ MPa, $E = 70$ GPa, Fig. 4–30*c*. If a force of $P = 60$ kN is applied to the rod and then removed, determine the residual stress in the rod.

SOLUTION

The free-body diagram of the rod is shown in Fig. 4–30*b*. Application of the load **P** will cause one of three possibilities, namely, both segments AC and CB remain elastic, AC is plastic while CB is elastic, or both AC and CB are plastic.*

An *elastic analysis*, similar to that discussed in Sec. 4.4, will produce $F_A = 45$ kN and $F_B = 15$ kN at the supports. However, this results in a stress of

$$\sigma_{AC} = \frac{45\text{ kN}}{\pi(0.005\text{ m})^2} = 573\text{ MPa (compression)} > \sigma_Y = 420\text{ MPa}$$

$$\sigma_{CB} = \frac{15\text{ kN}}{\pi(0.005\text{ m})^2} = 191\text{ MPa (tension)}$$

Since the material in segment AC will yield, we will assume that AC becomes plastic, while CB remains elastic.

For this case, the maximum possible force developed in AC is

$$(F_A)_Y = \sigma_Y A = 420(10^3)\text{ kN/m}^2\,[\pi(0.005\text{ m})^2] = 33.0\text{ kN}$$

and from the equilibrium of the rod, Fig. 4–31*b*,

$$F_B = 60\text{ kN} - 33.0\text{ kN} = 27.0\text{ kN}$$

The stress in each segment of the rod is therefore

$$\sigma_{AC} = \sigma_Y = 420\text{ MPa (compression)}$$

$$\sigma_{CB} = \frac{27.0\text{ kN}}{\pi(0.005\text{ m})^2} = 344\text{ MPa (tension)} < 420\text{ MPa (OK)}$$

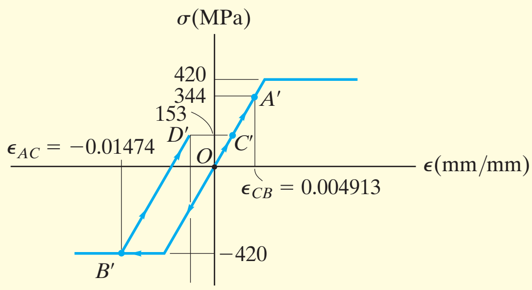
*The possibility of CB becoming plastic before AC will not occur because when point C moves, the *strain* in AC (since it is shorter) will always be larger than the strain in CB.

Residual Stress. In order to obtain the residual stress, it is also necessary to know the strain in each segment due to the loading. Since CB responds elastically,

$$\delta_C = \frac{F_B L_{CB}}{AE} = \frac{(27.0 \text{ kN})(0.300 \text{ m})}{\pi(0.005 \text{ m})^2[70(10^6) \text{ kN/m}^2]} = 0.001474 \text{ m}$$

$$\epsilon_{CB} = \frac{\delta_C}{L_{CB}} = \frac{0.001474 \text{ m}}{0.300 \text{ m}} = +0.004913$$

$$\epsilon_{AC} = \frac{\delta_C}{L_{AC}} = -\frac{0.001474 \text{ m}}{0.100 \text{ m}} = -0.01474$$



(c)

Fig. 4–30 (cont.)

Here the yield strain is

$$\epsilon_Y = \frac{\sigma_Y}{E} = \frac{420(10^6) \text{ N/m}^2}{70(10^9) \text{ N/m}^2} = 0.006$$

Therefore, when **P** is *applied*, the stress–strain behavior for the material in segment CB moves from O to A', Fig. 4–30*c*, and the stress–strain behavior for the material in segment AC moves from O to B'. If the load **P** is applied in the *reverse direction*, in other words, the load is removed, then an elastic response occurs and a reverse force of $F_A = 45$ kN and $F_B = 15$ kN must be applied to each segment. As calculated previously, these forces now produce stresses $\sigma_{AC} = 573$ MPa (tension) and $\sigma_{CB} = 191$ MPa (compression), and as a result the residual stress in each member is

$$(\sigma_{AC})_r = -420 \text{ MPa} + 573 \text{ MPa} = 153 \text{ MPa} \qquad \textit{Ans.}$$

$$(\sigma_{CB})_r = 344 \text{ MPa} - 191 \text{ MPa} = 153 \text{ MPa} \qquad \textit{Ans.}$$

This residual stress is the *same* for both segments, which is to be expected. Also note that the stress–strain behavior for segment AC moves from B' to D' in Fig. 4–30*c*, while the stress–strain behavior for the material in segment CB moves from A' to C' when the load is removed.